Exercise 1.3

Q.1 Which of the following are conformable for addition?

$A=\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$. $B=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$C=\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$, $D=\begin{bmatrix} 2+1 \\ 3 \end{bmatrix}$

$E=\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, $F=\begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$

Solution:
In the above matrices following matrices are suitable for addition.

(i) A and E are conformable for addition because their order is same and both are square matrix.

(ii) B and D are conformable for addition because the order is same i.e. they have two rows and 1 Columns and both are rectangular matrices.

(iii) C and F are conformable for addition because their order is same i.e. they have three 3 rows and 2 columns and they are a rectangular matrix.

Q.2 Find the additive inverse of the following matrices:

(1) $A=\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$

Solution:
Additive inverse of a matrix is negative matrix.

$A=\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$ is

$$-A=-\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}=\begin{bmatrix} (-1)\times 2 & (-1)4 \\ (-1)(-2) & (-1)1 \end{bmatrix}$$

$$-A=\begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$$

(2) $B=\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Solution: $B=\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Its additive inverse is

$$-B=-\begin{bmatrix} +1 & 0 & -1 \\ +2 & -1 & 3 \\ +3 & -2 & 1 \end{bmatrix}$$

$$-B=\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Solution: $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$-C = -\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -1\times 4 \\ -1\times -2 \end{bmatrix}$

The additive inverse is

$-C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$

(4) $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

Solution: $D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

The additive inverse is

$-D = -\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times -3 & -1\times -2 \\ -1\times 2 & -1\times 1 \end{bmatrix}$

$-D = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$

(5) $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution: $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The additive inverse of the given matrix is:

$-E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times 0 & -1\times 1 \end{bmatrix}$

$-E = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(6) $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Solution: $F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Its additive inverse is

$-F = -\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

$= \begin{bmatrix} -1\times\sqrt{3} & -1\times 1 \\ -1\times -1 & -1\times\sqrt{2} \end{bmatrix}$

$-F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$

Q.3 **If** $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$

$C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix},\ D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix},$

then find.

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution: $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

The order of matrix A and the given matrix order is same. So, they can be added easily.

$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

(ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Solution: $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

The order of both above matrices are same, so, they can be easily added.

$= \begin{bmatrix} 1+(-2) \\ -1+3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & +3 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

Solution: $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

As $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

Their orders are same so they can added

$= \begin{bmatrix} 1+(-2) & -1+(1) & 2+3 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & -1+1 & 5 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Solution: $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Their orders are same. So, they can be added.

$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

(v) $2A$

Solution: $2A$

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $2A$

$= (2) \times \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(-1) & 2\times 2 \\ 2\times 2 & 2\times 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

(vi) $(-1)B$

Solution: $(-1)B$

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $(-1)B$

$= (-1) \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} (-1)\times 1 \\ (-1)\times(-1) \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(vii) $(-2)C$

Solution: $(-2)C$

As $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, (-2) C

$= (-2) \times \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} (-2)(1) & (-2)(-1) & (-2)(2) \end{bmatrix}$

$= \begin{bmatrix} -2 & 2 & -4 \end{bmatrix}$

(viii) $3D$

Solution: $3D$

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $3D$

$= (3) \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3\times 1 & 3\times 2 & 3\times 3 \\ 3\times -1 & 3\times 0 & 3\times 2 \end{bmatrix}$

$$=\begin{bmatrix}3 & 6 & 9\\ -3 & 0 & 6\end{bmatrix}$$

(ix) $3C$

Solution: $3C$

As $C=\begin{bmatrix}1 & -1 & 2\end{bmatrix}$

So, 3C

$$=(3)\times\begin{bmatrix}1 & -1 & 2\end{bmatrix}$$
$$=\begin{bmatrix}3\times 1 & 3\times -1 & 3\times 2\end{bmatrix}$$
$$=\begin{bmatrix}3 & -3 & 6\end{bmatrix}$$

Q.4 Perform the indicated operations and simplify the following:

(i) $\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

Solution: $\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

$$=\begin{bmatrix}1+0 & 0+2\\ 0+3 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1+1 & 2+1\\ 3+1 & 1+0\end{bmatrix}$$
$$=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}$$

(ii) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\left(\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\right)$

Solution:

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\left(\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\right)$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0-1 & 2-1\\ 3-1 & 0-0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1-1 & 0+1\\ 0+2 & 1+0\end{bmatrix}$$
$$=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}$$

(iii) $\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\left(\begin{bmatrix}1 & 0 & 2\end{bmatrix}-\begin{bmatrix}2 & 2 & 2\end{bmatrix}\right)$

Solution:

$$=\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\begin{bmatrix}1-2 & 0-2 & 2-2\end{bmatrix}$$
$$=\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\end{bmatrix}$$
$$=\begin{bmatrix}2-1 & 3-2 & 1-0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

(iv) $\begin{bmatrix}1 & 2 & 3\\ -1 & -1 & -1\\ 0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & 3\end{bmatrix}$

Solution:

$$=\begin{bmatrix}1 & 2 & 3\\ -1 & -1 & -1\\ 0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & 3\end{bmatrix}$$
$$=\begin{bmatrix}1+1 & 2+1 & 3+1\\ -1+2 & -1+2 & -1+2\\ 3+0 & 1+2 & 2-1\end{bmatrix}$$
$$=\begin{bmatrix}2 & 3 & 4\\ 1 & 1 & 1\\ 3 & 4 & 5\end{bmatrix}$$

(v) $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 0\\ 0 & 2 & -1\end{bmatrix}$

Solution:

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 0\\ 0 & 2 & -1\end{bmatrix}$$
$$=\begin{bmatrix}1+1 & 2+1 & 3+1\\ -1+2 & -1+2 & -1+2\\ 3+0 & 1+2 & 2-1\end{bmatrix}$$
$$=\begin{bmatrix}2 & 2 & 1\\ 0 & 2 & 1\\ 3 & 3 & 1\end{bmatrix}$$

(vi) $\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

Solution:

$$=\begin{bmatrix}1+2 & 2+1\\0+1 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\1 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3+1 & 3+1\\1+1 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 4\\2 & 2\end{bmatrix}$$

Q.5 **For the matrices** $A=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$ **and** $C=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$, **verify the following rules:**

(i) $A+C=C+A$

Solutions:

L.H.S = A+C

R.H.S = C+A

LHS= A+C

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2+0 & 3+0\\2+0 & 3-2 & 1+3\\1+1 & -1+1 & 0+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

RHS= C+A

$$=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0+2 & 0+3\\0+2 & -2+3 & 3+1\\1+1 & 1-1 & 2-0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

$A+C=C+A$

Hence proved

L.H.S = R.H.S

(ii) $A+B=B+A$

Solution: $A+B=B+A$

L.H.S = A+B

R.H.S = B+A

LHS= A+B

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-1 & +3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

RHS= B+A

$$=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+3 & 1-1 & 3-0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\6 & 0 & 3\end{bmatrix}$$

$A+B=B+A$

Hence proved

L.H.S = R.H.S

(iii) $B+C=C+B$

Solution: $B+C=C+B$

L.H.S = B+C

R.H.S = C+B

L.H.S = B+C

$$=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1+0 & 1+0\\ 2+0 & -2-2 & 2+3\\ 3+1 & 1+1 & 3+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

R.H.S = C+B

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0-1 & 0+1\\ 0+2 & -2-2 & 3+2\\ 1+3 & 1+1 & 2+3\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & -4 & 5\\ 4 & 2 & 5\end{bmatrix}$$

L.H.S = R.H.S
B+C=C+B
Hence proved

(iv) $A+(B+A)=2A+B$

Solution: $A+(B+A)=2A+B$

L.H.S = A+ (B+A)
R.H.S = 2A+B
L.H.S= A+ (B+A)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

R.H.S = 2A+B

$$=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$$

L.H.S =R.H.S
A+ (B+A) =2A+B
Hence proved

(v) $(C-B)+A=C+(A+B)$

Solution: $(C-B)+A=C+(A+B)$

L.H.S = (C-B) +A
R.H.S = C+(A-B)
L.H.S = (C-B) + A

$$=\left(\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

RHS= C+ (A-B)

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 3 & 2\\ 0 & 5 & -1\\ -2 & -2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$$

L.H.S = R.H.S
(C-B)+A=C+ (A-B)
Hence proved

(vi) $2A+B=A+(A+B)$

Solution: $2A+B=A+(A+B)$

L.H.S = 2A+B
R.H.S = A+ (A+B)
LHS= 2A+B

$$=2\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}2&4&6\\4&6&2\\2&-2&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}3&3&7\\6&4&4\\5&-1&3\end{bmatrix}$$

RHS= A + (A+B)

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}$$

$$=\begin{bmatrix}3&3&7\\6&4&4\\5&-1&3\end{bmatrix}$$

L.H.S = R.H.S

2A+B=A+ (A+B)

Hence proved

(vii) $(C-B)-A=(C-A)-B$

Solution: $(C-B)-A=(C-A)-B$

L.H.S = (C-B) –A

R.H.S = (C-A) –B

LHS= (C-B) – A

$$=\left(\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)-\begin{bmatrix}1&2&3\\2&3&1\\1&-&0\end{bmatrix}$$

$$=\begin{bmatrix}-2&1&-1\\-2&0&1\\-2&0&-1\end{bmatrix}-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}-3&-1&-4\\-4&-3&0\\-3&1&-1\end{bmatrix}$$

RHS= (C-A) –B

$$=\left(\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}\right)-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}-2&-2&-3\\-2&-5&2\\0&2&2\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}-3&-1&-4\\-4&-3&0\\-3&1&-1\end{bmatrix}$$

L.H.S = R.H.S

(C-B)-A=(C-A)-B

Hence proved

(viii) $(A+B)+C=A+(B+C)$

Solution: (A+B) + C = A+ (B+C)

L.H.S = (A+B) + C

R.H.S = A + (B+C)

LHS= (A+B) +C

$$=\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$=\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$=\begin{bmatrix}1&1&4\\4&-1&6\\5&1&5\end{bmatrix}$$

R.H.S = A+ (B+C)

$$=\begin{bmatrix}1&2&3\\2&3&1\\3&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}\right)$$

$$=\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1 & 4\\ 4 & -1 & 6\\ 5 & 1 & 5\end{bmatrix}$$

L.H.S = R.H.S

(A+B)+C=A+ (B+C)

Hence proved

(ix) $A+(B-C)=(A-C)+B$

Solution: $A+(B-C)=(A-C)+B$

L.H.S = A+ (B-C)

R.H.S = (A-C) + B

L.H.S = A+ (B-C)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & -1 & 1\\ 2 & 0 & -1\\ 2 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$$

RHS= (A-C) +B

$$=\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right)+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-0 & 3-1\\ 2-0 & 3+2 & 1-3\\ 1-1 & -1-1 & 0-2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2 & 3\\ 2 & 5 & -2\\ 0 & -2 & -2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$$

L.H.S = R.H.S

A+ (B-C) = (A-C) +B

Hence proved

(x) $2A+2B=2(A+B)$

Solution: $2A+2B=2(A+B)$

L.H.S = 2A + 2B

R.H.S = 2(A + B)

L.H.S = 2A + 2B

$$=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+2\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}2 & -2 & 2\\ 4 & -4 & 4\\ 6 & 2 & 6\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 8\\ 8 & 2 & 6\\ 8 & 0 & 6\end{bmatrix}$$

RHS= 2 (A+B)

$$=2\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=2\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 8\\ 8 & 2 & 6\\ 8 & 0 & 6\end{bmatrix}$$

L.H.S = R.H.S

2A+2B=2(A+B)

Hence proved

Q.6 **If** $A=\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$ **and** $B=\begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$

find:

(i) $3A-2B$

Solution: $3A-2B$

$$3A-2B=3\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}-2\begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$$

$$=\begin{bmatrix}3 & -6\\ 9 & 12\end{bmatrix}-\begin{bmatrix}0 & 14\\ -6 & 16\end{bmatrix}$$

$$=\begin{bmatrix}3 & -20\\15 & -4\end{bmatrix}$$

(ii) $2A^t-3B^t$

Solution: $2A^t-3B^t$

When we take transpose of any matrix we change rows into columns or columns into rows.

$$A^t=\begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}0 & -3\\7 & 8\end{bmatrix}$$

$$2A^t-3B^t=2\begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix}-3\begin{bmatrix}0 & -3\\7 & 8\end{bmatrix}$$

$$=\begin{bmatrix}2 & 6\\-4 & 8\end{bmatrix}-\begin{bmatrix}0 & -9\\21 & 24\end{bmatrix}$$

$$=\begin{bmatrix}2 & 15\\-25 & -16\end{bmatrix}$$

Q.7 If

$$2\begin{bmatrix}2 & 4\\-3 & a\end{bmatrix}+3\begin{bmatrix}1 & b\\8 & -4\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

Solution:

$$2\begin{bmatrix}2 & 4\\-3 & a\end{bmatrix}+3\begin{bmatrix}1 & b\\8 & -4\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 8\\-6 & 2a\end{bmatrix}+\begin{bmatrix}3 & 3b\\24 & -12\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$$=\begin{bmatrix}7 & 8+3b\\18 & 2a+(-12)\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$8 + 3b = 10$ __________ (i)
$2a - 12 = 1$ __________ (ii)

By solving equation (ii) we get the value of a

$2a - 12 = 1$
$2a = 1 + 12$
$2a = 13$
$a=\frac{13}{2}$

By solving equation (i) we get the value of b

$8 + 3b = 10$
$3b = 10 - 8$
$3b = 2$
$b=\frac{2}{3}$

Q.8 **If** $A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ **and** $B=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}$

Then verify that

(i) $(A+B)^t=A^t+B^t$

Solution: $(A+B)^t=A^t+B^t$

L.H.S = $(A + B)^t$
R.H.S = $A^t + B^t$
To solve L.H.S
L.H.S = $(A + B)^t$

$$=(A+B)^t=\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$$

$$=\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}^t$$

$$\text{R.H.H}=(A+B)^t=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$

To solve R.H.S
R.H.S = $A^t + B^t$

$$A^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$\text{RHS}=A^t+B^t=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}+\begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$

L.H.S = R.H.S $\Rightarrow (A+B)^t=A^t+B^t$
Hence Proved

(ii) $(A-B)^t=A^t-B^t$

Solution: $(A-B)^t=A^t-B^t$

L.H.S = $(A - B)^t$
R.H.S = $A^t - B^t$
LHS= $(A - B)^t$

$$(A-B)^t = \left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}-\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$$

$$=\begin{bmatrix}0 & 1\\-2 & 1\end{bmatrix}^t$$

$$(A-B)^t = \begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$$

R.H.S = $A^t - B^t$

$$=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$

$$=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 0-2\\2-1 & 1-0\end{bmatrix}$$

$$=\begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$$

L.H.S = R.H.S

$(A-B)^t = A^t - B^t$

Hence proved

(iii) $A + A^t$ **is a symmetric**

Solution:

$A + A^t$ is a symmetric

To show that $A + A^t$ is symmetric, we will show that

$$(A+A^t)^t = (A+A^t)$$

$$A+A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$$

$$=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2+0\\0+2 & 1+1\end{bmatrix}$$

$$A+A^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

$$(A+A^t)^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}^t$$

$$=\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

$(A+A^t)^t = (A+A^t)$

Hence Proved

$A + A^t$ symmetric

(iv) $A - A^t$ **is a skew symmetric**

Solution: $A - A^t$

To show that $A - A^t$ is skew symmetric we will show that

$$(A-A^t)^t = -(A-A^t)$$

$$A - A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t - \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$$

$$=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2-0\\0-2 & 1-1\end{bmatrix}$$

$$A - A^t = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

$$(A-A^t)^t = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}^t$$

$$=\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$$

$$=-\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

$(A-A^t)^t = -(A-A^t)$

Hence proved

$A - A^t$ is a skew symmetric

(v) $B + B^t$ **is a symmetric**

Solution: $B + B^t$

The show that $B+B^t$ is symmetric we will show that

$$(B+B^t)^t = (B+B^t)$$

$$B + B^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$

$$=\begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 1+2\\2+1 & 0+0\end{bmatrix}$$

$$B + B^t = \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}$$

$$(B+B^t)^t = \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}^t$$

$$=\begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}$$

$$(B+B^t)^t = (B+B^t)$$

Hence proved
$B + B^t$ is a symmetric

(vi) $B - B^t$ **is a skew symmetric**
Solution: $B - B^t$
To show that B – B^t is skew symmetric, we will show that

$(B-B^t)^t = -(B - B^t)$

$$B - B^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 1-2 \\ 2-1 & 0-0 \end{bmatrix}$$

$$B - B^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(B - B^t)^t = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$- \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$(B - B^t)^t = -(B - B^t)$

Hence proved
$B - B^t$ is a skew symmetric.